Probability and Random Processes

ECS 315

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7 Random Variables

Office Hours:

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Wednesday 14:30-15:30

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Chapter 5 vs. Chapter 7

- Chapter 5: Finding **probability of an event**
  Before the midterm, we studied how to find the probability of any event A by adding the probabilities of the outcomes inside A.
  - Ex. When $A = \{a, b\}$, we can calculate the probability of A by
    $$P(A) = P(\{a, b\}) = P(\{a\}) + P(\{b\})$$
- Chapter 7: Finding probability involving **a random variable**

Review: An example in Chapter 5

$P(\{a\}) = 0.1$ $P(\{b\}) = 0.3$ $P(\{c\}) = 0.5$

Example 5.7. A random experiment can result in one of the outcomes $\Omega = \{a, b, c, d\}$ with probabilities 0.1, 0.3, 0.5, and 0.1, respectively. Let A denote the event $\{a, b\}$, B the event $\{b, c, d\}$, and C the event $\{d\}$.

$P(\{d\}) = 0.1$

$\{a,b\} = \{a\} \cup \{b\}$ disjoint Finite additivity

- $P(A) = P(\{a,b\}) = P(\{a\} \cup \{b\}) = P(\{a\}) + P(\{b\}) = 0.1 + 0.3 = 0.4$
- $P(B) = P(\{b,c,d\}) = P(\{b\} \cup \{c\} \cup \{d\}) = P(\{b\}) + P(\{c\}) + P(\{d\}) = 0.3 + 0.5 + 0.1 = 0.9$

  disjoint Finite additivity
- $P(C) = P(\{d\}) = 0.1$
- $P(A^c) = P(\{c,d\}) \overset{(5.6b)}{=} P(\{c\}) + P(\{d\}) = 0.5 + 0.1 = 0.6$

Review: Steps we used in CH5

To find the probability of an event:

1. Identify the sample space Ω and the probability $P(\{\omega\})$ for each outcome ω.

   Example 5.7. A random experiment can result in one of the outcomes $\{a, b, c, d\}$ with probabilities 0.1, 0.3, 0.5, and 0.1, respectively. Let A denote the event $\{a, b\}$, B the event $\{b, c, d\}$, and C the event $\{d\}$.

   $P(\{a\}) = 0.1$, $P(\{b\}) = 0.3$, $P(\{c\}) = 0.5$, $P(\{d\}) = 0.1$

2. Identify all the outcomes inside the event under consideration.

   $$P(A^c) = P(\{c,d\}) = P(\{c\}) + P(\{d\}) = 0.5 + 0.1 = 0.6$$

3. Add the probability $P(\{\omega\})$ of the outcomes from the previous step.

Chapter 5 vs. Chapter 7

- Chapter 5: Steps to find the **probability of an event**
  1. Identify the sample space Ω and the probability $P(\{\omega\})$ for each outcome ω.
  2. Identify all the outcomes inside the event under consideration.
  3. Add the probability $P(\{\omega\})$ of the outcomes from the previous step.
- Chapter 7: Steps to find **probability involving RV**

  ?

Chapter 7

- Crucial Skill 7.1: Find probability involving RV when the RV is defined as a function of outcomes

Steps to find probability involving RV

when the RV is defined as a function of outcomes

Ex. $X(\omega) = \omega$

$Y(\omega) = (\omega - 3)^2$

$Z(\omega) = \sqrt{Y(\omega)}$

Usually given as a statement about the RV

Ex. $X > 3$

$X = 3$

$|X| < 2$

1. Identify the sample space Ω and the probability $P(\{\omega\})$ for each outcome ω.
2. Consider the given statement. Find the values of ω that make the RV satisfy the given statement.
   - To do this, consider the statement, substitute the RV in the statement by its definition, and solve for ω.
3. Add the probability $P(\{\omega\})$ of the outcomes from the previous step.

Let's call this "Method 1"

Example

- Roll a fair dice. Let $\Omega = \{1,2,3,4,5,6\}$.
- Define $Y(\omega) = (\omega - 3)^2$. Find $P[Y = 4]$.
- Ω is given. The dice is fair; therefore and the probability $P(\{\omega\}) = \frac{1}{6}$ for each outcome ω inside Ω.

Method 1:

The statement under consideration is "$Y = 4$".

From $Y(\omega) = (\omega - 3)^2$, $Y(\omega) = 4$ occurs when $\omega = 1$ or 5.

Therefore, $P[Y = 4] = P(\{1\}) + P(\{5\}) = \frac{2}{6} = \frac{1}{3}$

The connection between Chapter 5 and Chapter 7

- Probability involving RV is expressed in the form

$$P[\text{some statement(s) about } X]$$

- Technically, when we write

$$[\text{some statement(s) about } X],$$

we are actually defining an event

$A =$ the event containing outcomes ω that make $X(\omega)$ satisfy the given statement

- Now that we have an event, we can apply the steps in Chapter 5 to find $P(A)$.

Let's call this "Method 2"

Example

- Roll a fair dice. Let $\Omega = \{1,2,3,4,5,6\}$.
- Define $Y(\omega) = (\omega - 3)^2$. Find $P[Y = 4]$.
- Ω is given. The dice is fair; therefore and the probability $P(\{\omega\}) = \frac{1}{6}$ for each outcome ω inside Ω.

Method 1:

The statement under consideration is "$Y = 4$".

From $Y(\omega) = (\omega - 3)^2$, $\quad Y(\omega) = 4$ occurs when $\omega = 1$ or 5.

Therefore, $P[Y = 4] = P(\{1\}) + P(\{5\}) = \frac{2}{6} = \frac{1}{3}$

Method 2:

$$[Y = 4] = \{\omega: Y(\omega) = 4\} = \{\omega: (\omega - 3)^2 = 4\} = \{1,5\}$$

$$P[Y = 4] = P([Y = 4]) = P(\{1,5\}) = P(\{1\}) + P(\{5\}) = \frac{2}{6} = \frac{1}{3}$$

Chapter 7

- Crucial Skill 7.1: Find probability involving RV when the RV is defined as a function of outcomes.
- Skill 7.2: Know the difference between X and x.
- Crucial Skill 7.3: Determine whether a set is a support of a RV.